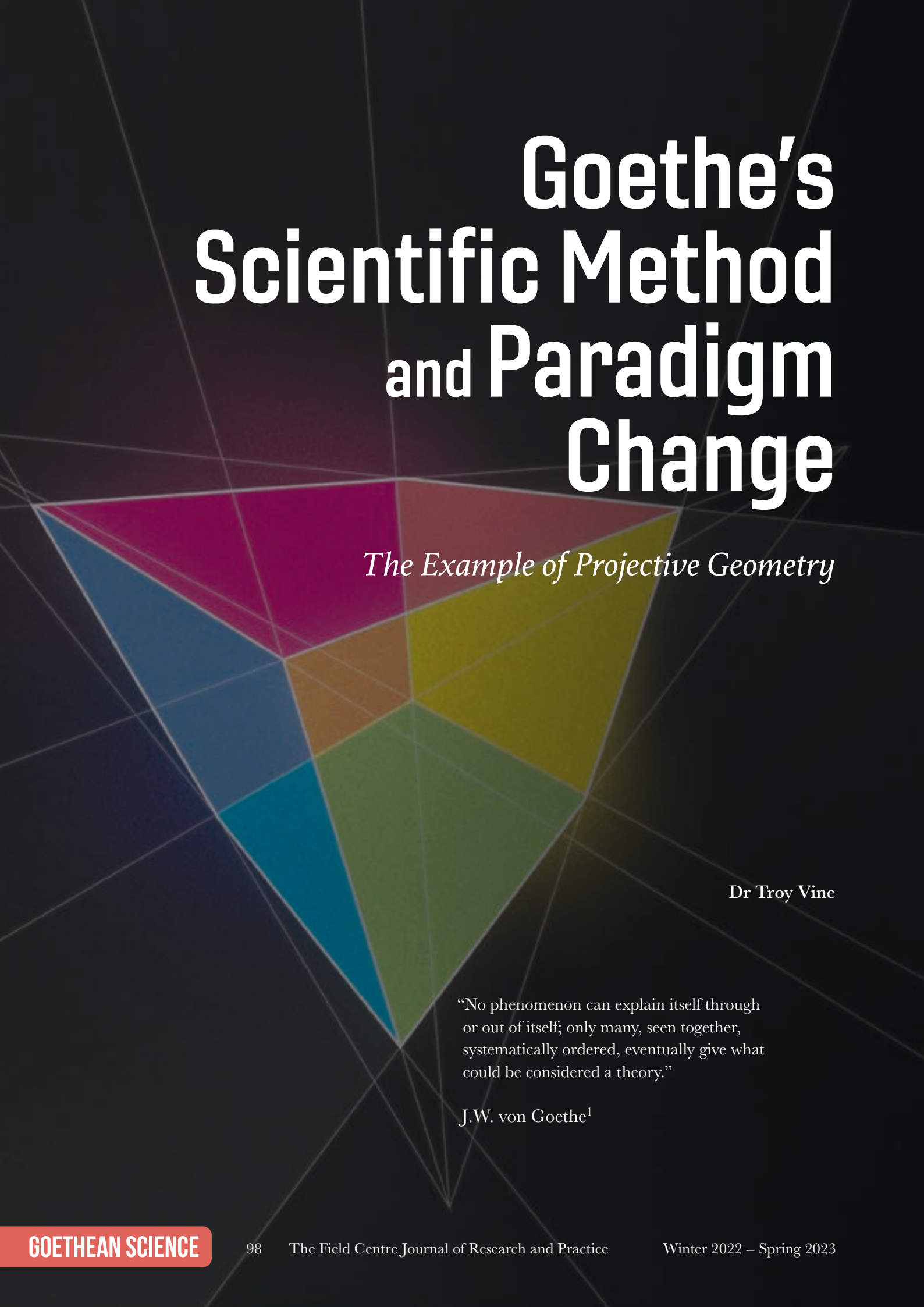




Goethe's Scientific Method and Paradigm Change

The Example of Projective Geometry

Dr Troy Vine

“No phenomenon can explain itself through
or out of itself; only many, seen together,
systematically ordered, eventually give what
could be considered a theory.”

J.W. von Goethe¹

I. Introduction

In “The Metamorphosis of the Scientist”, Frederick Amrine claims that Goethe not only “anticipates many of the central tenets of the most recent philosophy of science, he also offers solutions to its central dilemmas in both theory and practice”.² The central tenets Amrine has in mind in this bold claim are those of Thomas Kuhn’s conception of a scientific paradigm presented in *The Structure of Scientific Revolutions*; the dilemmas are that this work “cannot account for the structure of scientific revolutions because they are ultimately unstructured and extra-scientific”.³ Remarkably, despite being written three decades ago, Amrine’s essay remains one of the few attempts to bring Goethe’s scientific studies and Kuhn’s philosophy of science into dialogue. It thus provides a welcome starting point for a promising endeavour that will be taken up here.

In his account of scientific revolutions, Kuhn remarks that scientists receive a “narrow and rigid education”.⁴ As a result, “scientific training is not well designed to produce a man who will easily discover a fresh approach”.⁵ Because of the rigidity of the individual scientists working within a paradigm, a new paradigm is generally introduced by a young scientist or someone new to the field. For Kuhn, then, there is no real need to develop a method or training for paradigm change. For while the individuals working within a particular paradigm will generally not switch paradigms, the community as a whole does by training aspiring scientists according to the new paradigm: “Given a generation in which to effect the change, individual rigidity is compatible with a community that can switch from paradigm to paradigm when the occasion demands”.⁶

This is precisely here where Amrine locates Goethe’s scientific contribution. By training the scientist “*not* to reduce phenomena to a scheme but, rather, to remain inwardly mobile”,⁷ Goethe’s method provides “the ground from

This article is based on a lecture and workshop given at the Field Centre in 2022. In addition to participants of these events, for helpful comments I would like to thank Charles Gunn, Olaf Müller, Thomas Raysmith, Michael Beaney and his students.

- 1 Johann Wolfgang von Goethe, *Hamburger Ausgabe*, vol. 12 (München: Beck, 1981), 434. This edition will be abbreviated to HA in subsequent citations. All translations from German are my own.
- 2 Frederick Amrine, “The Metamorphosis of the Scientist”, in *Goethe’s Way of Science*, edited by David Seamon and Arthur Zajonc (Albany, NY: State University of New York Press, 1998), 36.
- 3 Amrine, “The Metamorphosis of the Scientist”, 35.
- 4 Thomas S. Kuhn, *The Structure of Scientific Revolutions* (Chicago: University of Chicago Press, 2012), 165.
- 5 Kuhn, *Structure*, 165.
- 6 Kuhn, *Structure*, 165.
- 7 Amrine, “The Metamorphosis of the Scientist”, 38.

which all our *Vorstellungsarten* – all scientific ‘paradigms’ – arise”.⁸ Goethe focused his scientific activity on botany and chromatics “above all because of their propaedeutic value”.⁹ The propaedeutic value of these examples is to lead the scientist to what “moves *between* the paradigms, the source of the revolutions themselves” and thereby develop the “mobility of one’s thinking and imagination, upon which the ‘metamorphosis of the scientist’ and, with it, the growth of science depend”.¹⁰

In *The Metamorphosis of Plants*, Goethe gives an account of the growth of the plant by showing how a given stage in its development is the metamorphosis of the previous stage.¹¹ But Amrine gives no such account of the growth of science. His response to the question of how “can there be progress in science in any real sense”¹² is the development “in a rigorous and controlled manner [of] new ways of seeing”.¹³ But this can only be half the answer. Amrine does not show how developing a new way of seeing can bring about a *progression* from one paradigm to the next, rather than merely *replacing* one paradigm with another. In short, what is missing is an account of the metamorphosis of the paradigm. In the following I present an example of the paradigm change from Euclidian to projective geometry in order to exemplify Goethe’s method and thereby show how a scientific revolution brings about just such a metamorphosis of the paradigm.

2. Goethe’s Scientific Method

In the final chapter of the three-volume *Farbenlehre*, published in 1810, Goethe describes his experience of looking through a prism that led to his rejection of the Newtonian paradigm in colour science. He recounts that he wrote his *Contributions to Optics*, the first part of which was published in 1791, in order to “produce the *aperçu* in another that had had such a vivid effect on me”.¹⁴ In short, Goethe’s intention was to bring about what Kuhn calls a “scientific revolution”.¹⁵

The method employed in *Contributions to Optics* is outlined in “The Experiment as Mediator between Object and Subject”, written in 1792. Goethe concludes this essay with the following methodological summary:

8 Amrine, “The Metamorphosis of the Scientist”, 47.

9 Amrine, “The Metamorphosis of the Scientist”, 44.

10 Amrine, “The Metamorphosis of the Scientist”, 47.

11 HA 13: 64–101.

12 Amrine, “The Metamorphosis of the Scientist”, 34.

13 Amrine, “The Metamorphosis of the Scientist”, 36.

14 HA 14: 264.

15 See Nora Löbe, Matthias Rang and Troy Vine, *Seeing Colour: A Journey Through Goethe’s World of Colour* (Edinburgh: Floris Books, 2022), 12–15.

My intention is to collect all experiences [*Erfahrungen*] in this subject, carry out all experiments myself and do so in their greatest manifoldness [*Mannigfaltigkeit*] so that they become easy to reproduce and are not inaccessible to so many people; then to establish the theorems [*Sätze*] in which the experience of a higher kind can express itself and see if they can be brought under a higher principle.¹⁶

We can discern three stages in this method, namely 1) manifolding phenomena, 2) establishing theorems and 3) bringing them under a higher principle.¹⁷ And although he does not say anything about the third stage other than it is a “higher principle”, it is clear from the experimental presentation in *Contributions to Optics* that this higher principle is that of polarity.¹⁸

It remains unclear, however, what distinguishes the theorems of stage two from the higher principle of stage three – polarity is a good candidate for both. In the *Farbenlehre* we find the same three methodological stages, with the theorems of stage two described as “scientific categories [*Rubriken*] that point further upwards”.¹⁹ These are subsumed in stage three “under higher rules and laws that reveal themselves [...] as phenomena to seeing [*Anschauung*]”.²⁰ Goethe calls such phenomena “archetypal phenomena”.²¹ Here, instead of “theorems” and “higher principles” we have “scientific categories” and “rules and laws”, but this does not clarify what distinguishes them.

These methodological descriptions show that Goethe’s method has three distinct stages, which he also calls “the empirical phenomenon”, “the scientific phenomenon” and “the pure phenomenon” respectively.²² In order to gain some clarity on what distinguishes the second and third stage, it is instructive to consider a concrete example.

16 HA 13: 20. Goethe’s mathematical analogy here is precise; in mathematics, theorems (*Sätze*) are derived from higher principles, i.e. axioms or postulates.

17 I disagree here with Eckart Förster that the methodological description in “The Experiment as Mediator” has only two stages. See *Die 25 Jahre der Philosophie: Eine systematische Rekonstruktion* (Frankfurt am Main: Klosterman, 2012), 128–129.

18 For an overview of these experiments and how they show the principle of polarity, see Troy Vine, “Newton, Goethe and the Mathematical Style of Thinking: A Critique of Henri Bortoft’s *Taking Appearance Seriously*”, *In Dialogue* 1 (2020): 83–86.

19 HA 13: 368.

20 HA 13: 368.

21 HA 13: 368. The shift from the singular “higher principle” before to the plural here is because Goethe now sees two higher principles expressed in colour phenomena, namely polarity and enhancement.

22 HA 13: 25.

3. The Polarity of Space

To exemplify Goethe's method we will consider an example that illustrates the paradigm change from Euclidean to projective geometry. Goethe's *Contributions to Optics* is not a historical account of his own paradigm change, but rather, as Amrine notes, a presentation chosen for its "propaedeutic value".²³ The example we will consider here, taken from German mathematician Immo Diener's introduction to projective geometry, is similarly didactic rather than historical in nature.²⁴

3.1 The Empirical Phenomenon

Diener begins his introduction by observing a cube and an octahedron, shown in figure 1.

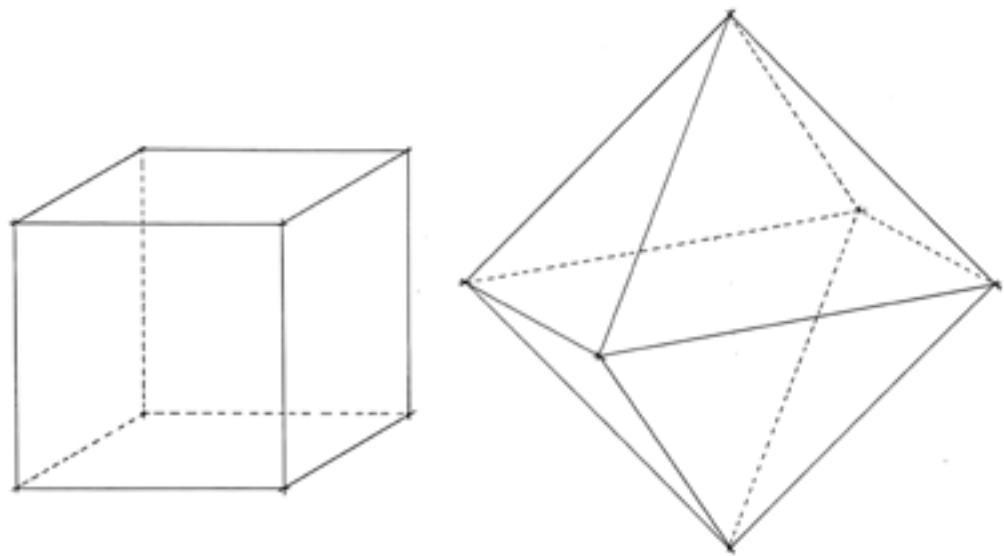


Figure 1. Cube and octahedron.

Goethe describes the first stage as the "manifolding of each experiment",²⁵ so we begin by collecting observations. Statements about the cube are listed on the left, corresponding statements about the octahedron on the right:²⁶

23 Amrine, "The Metamorphosis of the Scientist", 42, 44.

24 This section is a summary of Immo Diener's introduction in *Projektive Geometrie: Denken in Bewegung: Eine Epoche in der 11. Klasse* (Stuttgart: Pädagogische Forschungsstelle, 2017), 11–38.

25 HA 13: 18.

26 Diener, *Projektive Geometrie*, 18.

There are 6 faces, 8 vertices, 12 edges.

Each vertex has 3 edges.

Each face has 4 edges.

Each vertex has 3 faces.

Each face has 4 vertices.

The face with which a given face has no edge in common is the opposite face.

? (Each point has 4 such points.)

There are 8 faces, 6 vertices, 12 edges.

Each vertex has 4 edges.

Each face has 3 edges.

Each vertex has 4 faces.

Each face has 3 vertices.

? (Each face has 4 such faces.)

The vertex with which a given vertex has no edge in common is the opposite vertex.

This list would be greatly expanded in the first stage of investigation, but suffices to illustrate the general point. Characteristic of the first stage is that “no consistent systematic can be seen in the sense that one immediately knows what will be on the other side when one knows what is on one side”.²⁷ Also, as Diener notes, “for some of the observations there is nothing corresponding on the other side”.²⁸

3.2 The Scientific Phenomenon

In the list above we could not see a connection between a statement on one side and the corresponding statement on the other. However, if we compare statements in which the same numbers appear we might notice a connection between statements about faces and statements about vertices.

Based on this correspondence, we could rearrange the right-hand column of our list as follows:²⁹

There are 6 faces, 8 vertices, 12 edges.

Each vertex has 3 edges.

Each face has 4 edges.

Each vertex has 3 faces.

Each face has 4 vertices.

The face with which a given face has no edge in common is the opposite face.

There are 6 vertices, 8 faces, 12 edges.

Each face has 3 edges.

Each vertex has 4 edges.

Each face has 3 vertices.

Each vertex has 4 faces.

The vertex with which a given vertex has no edge in common is the opposite vertex.

²⁷ Diener, *Projektive Geometrie*, 21.

²⁸ Diener, *Projektive Geometrie*, 21.

²⁹ Diener, *Projektive Geometrie*, 22.

When our observations are placed side by side in this manner we can see that a statement on one side can be transformed into a corresponding statement on the other by interchanging the terms “vertex” and “face”. The term “edge”, however, does not change under such a transformation.³⁰ Also, all statements on one side now have a corresponding statement on the other.

We can see that each transformation is the expression of a “theorem” – we have reached the second stage of Goethe’s method. The polarity of point and plane seems a good candidate for what Goethe characterises as “scientific categories that point further upwards”.

We can continue the process of manifolding and add increasingly more complex observations. The edges and faces of the solids can be extended as lines and planes. The further geometrical relationships these extended lines and planes produce can also be included in our observations. For example:³¹

There are 8 planes in space in which lie exactly 3 vertices of the cube.

These 8 planes form the faces of an octahedron, which lies inside the cube.

There are 8 points in space that lie in exactly 3 planes of the octahedron.

These 8 points form the points of a cube, which lies outside the octahedron.

These last two statements, shown in figure 2, reveal an incredible polarity between not only the two solids themselves but the space in which they are embedded – notice how “inside” transforms to “outside” and *vice versa*.

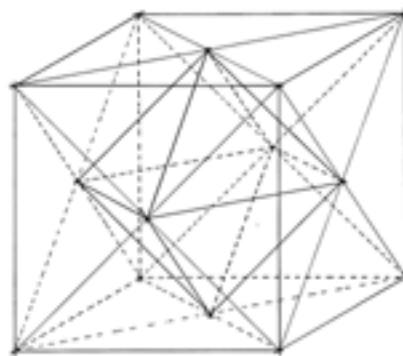


Figure 2. The 8 planes in each of which lie 3 vertices of the cube form the faces of an octahedron. Conversely, the 8 points each of which lie in 3 planes of the octahedron form the vertices of a cube.

³⁰ Mathematicians usually call this symmetry between points, lines and planes – i.e. three elements – a “duality”. I am calling it a “polarity” to distinguish it from the two-dimensional case, which only has points and lines – two elements – that are *stricto sensu* dual.

³¹ Diener, *Projektive Geometrie*, 26.

After making more observations that correspond in this manner, we might be tempted to conclude that our scientific category has now become the higher principle – i.e. we have reached stage three and our Goethean observation is thus complete. We could express this principle by unifying the two solids into a single solid, shown in figure 3, in which each element of one original solid is uniquely related to an element of the other in a fully polar way. We might then claim that this solid is the archetypal phenomenon.

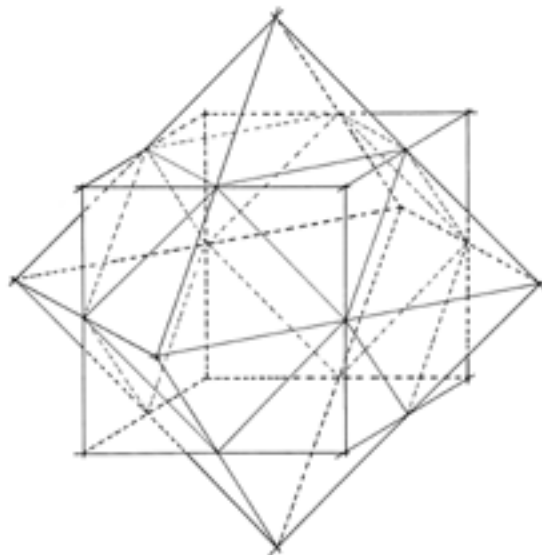


Figure 3. The cube and octahedron unified into a single solid such that each element of one original solid is uniquely related to one element of the other in a fully polar way.

What is instructive about this particular example is that such a conclusion would be premature. If we continue manifolding our observations, we will sooner or later end up with the following:³²

The 12 edges form 3 groups of 4 lines that are parallel to each other.	Not true!
Not true!	The 12 edges form 6 groups of 2 lines that are parallel to each other.

Through the process of manifolding, we have discovered observations that do not appear to express the polarity seen so far; we have found statements that are not true of both solids, because, according to our substitution rules, edges remain edges.

32 Diener, *Projektive Geometrie*, 25.

The concept of parallelism breaks the symmetry between cube and octahedron. We are thus able to reach stage two of Goethe's method, but unable to progress to stage three. There is an element of our conceptual system – the parallel postulate – that prevents polarity from being a higher principle. At best we could say that there is a partial polarity between the two solids, but it would actually be correct to say that what looked like a polarity turned out not to be a polarity after all.

We have now reached what I consider to be the crux of the transition from stage two to stage three. If we want to see a particular domain – e.g. geometry – as the expression of a higher principle – e.g. polarity – we have to transform our conceptual system, we have to change the paradigm. Diener's basic idea to bring about this transformation and progress from the second to the third stage is to hold on to the idea of polarity and explore the implications – i.e. see whether what we have discovered so far can be subsumed under "higher rules and laws".

Diener's basic idea can be illustrated in the following way. First, we omit the word "parallel" from the problematic statement about the cube:

The 12 edges form 3 groups of 4 edges that...	?
---	---

We then see if we can find a corresponding statement about the octahedron. Considering the planes that pass through four vertices gives us:

The 12 edges form 3 groups of 4 lines that...	The 12 edges form 3 groups of 4 lines that <i>lie in a plane</i> .
---	--

This, in turn, allows us to complete the statement for the cube:

The 12 edges form 3 groups of 4 lines that <i>meet at a point</i> .	The 12 edges form 3 groups of 4 lines that lie in a plane.
---	--

This shows that holding on to the idea of polarity entails a conceptual transformation resulting in parallel lines that pass through a point. But where is this point? Proceeding in the same manner as before, we can extend the statement about the lines of the octahedron on the right-hand side:

The 12 edges form 3 groups of 4 lines that meet at a point *and the 3 points lie on the plane at...*

The 12 edges form 3 groups of 4 lines that lie in a plane *and the 3 planes meet at the point in the centre.*

It remains unclear what the counterpart is to the point at the centre. We have seen above that “inside” transforms to “outside”, so this plane must be outside the solid, but where? Although we might have an idea, we cannot *show* where it is proceeding as we have been.

3.3 The Archetypal Phenomenon

At this stage, Diener introduces movement into the process by taking the image in figure 3 and moving it in our imagination.³³ In what he calls “the great metamorphosis”,³⁴ we view with our mind’s eye the polar movement that occurs when we take the unified solid shown in figure 3 and contract first the octahedron considered as points (first statement, right-hand side) and then the cube considered as planes (second statement, left-hand side):³⁵

The 6 planes expand to a single plane at the periphery.

The 6 points contract to a single point at the centre.

The 6 planes contract to 3 planes that intersect at the point at the centre.

The 6 points expand to 3 points that lie on the plane at the periphery.

These movements thus reveal further features of the polarity of point and plane: the polar movement of the contraction of one solid to a point at the centre is the expansion of the other to the plane at the periphery. Moreover, two points moving towards each other and forming a single point at the centre is polar to two planes moving away from each other and forming a single plane at the periphery. Conversely, two planes moving towards each other and forming a single plane at the centre is polar to two points moving away from each other and forming a single point at the periphery.

The movements of expansion and contraction described above are better thought of as two aspects of a single movement. If we consider the movements of the points of the octahedron, we can see that they actually move in a straight line throughout the whole expansion and contraction. When the

33 For an insightful study of the use of imagination in Goethe’s method, see Dennis Sepper, “The Farbenlehre and Goethe’s Nonromantic Scientific Imagination”, *In Dialogue 2* (2022): 56–71.

34 Diener, *Projektive Geometrie*, 51.

35 Diener’s more detailed account of this movement and two preliminary exercises that I have omitted due to lack of space is in *Projektive Geometrie*, 39–53.

octahedron contracts, opposite points move towards each other, pass each other – momentarily becoming a single point at the centre – and continue in the same direction when the octahedron expands. They then pass each other again – momentarily becoming a single point at the periphery – and continue in the same direction as the octahedron contracts again, which is now towards the centre from the opposite side. The same is true for the planes of the expanding and contracting cube.

We can now see that it is not the unified solid shown in figure 3 but its polar movement that is the archetypal phenomenon of space. It is archetypal because it is not dependent on the cube and octahedron. Indeed, it can be done with any solid and its polar counterpart. The only reason for using the cube and the octahedron is the propaedeutic value.

Having found the archetypal phenomenon of space, we can now apply our understanding to the transformation of the problematic statements about the cube and octahedron in the second stage. As we have seen, the plane that is polar to the point at the centre is at the periphery. Completing our partial transformation for the cube gives the following:

The 12 edges form 3 groups of 4 lines that meet at a point and the 3 points lie on the plane *at the periphery*.

The 12 edges form 3 groups of 4 lines that lie in a plane and the 3 planes meet at the point in the centre.

We can see these three points on the plane at the periphery if we redraw the cube in perspective, shown in figure 4.

We can also transform the problematic statement about the octahedron in a similar manner, giving the statement on the right-hand side:

The 12 edges form 6 groups of 2 lines that lie in a plane and the 6 planes meet at the point in the centre.

The 12 edges form 6 groups of 2 lines that meet at a point and the 6 points lie on the plane *at the periphery*.

The transformed statement about the octahedron on the right-hand side now has a corresponding statement about the cube on the left-hand side that is also true. By transforming the conceptual system, the two seemingly corresponding, problematic statements about the cube and the octahedron have been resolved into two separate statements, each with its own polar statement.

4. Paradigm Change

Using the example in the previous section we can characterise the three stages of Goethe's method as follows:

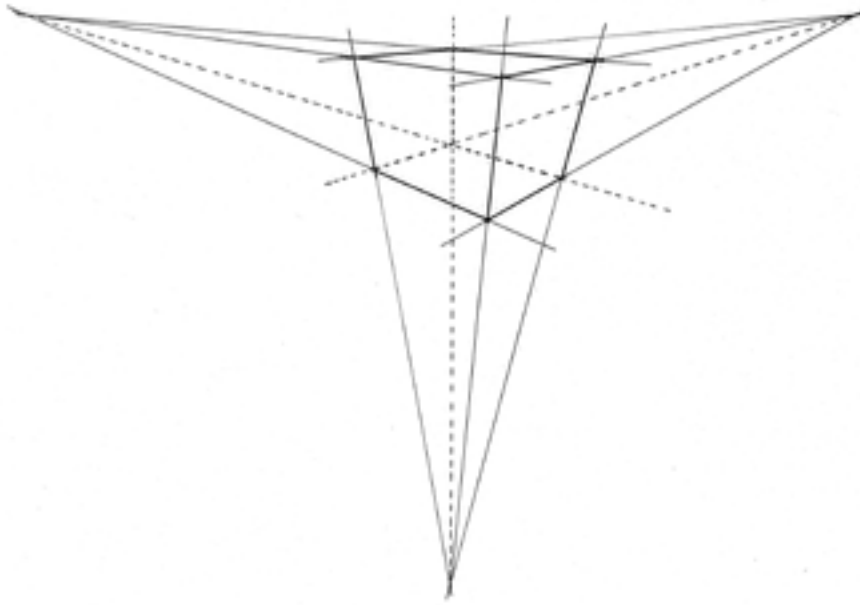


Figure 4. A cube drawn in perspective showing the three points at which the parallel lines meet at the peripheral plane.

- 1) In the first stage we collect observations within a particular domain. We transition to the second stage when we see a lawfulness to our observations, in this case a polarity between point and plane.
- 2) In the second stage we look systematically for instances of this lawfulness. In doing so we also become aware of the paradigm in which we are operating. In particular, at some point we find that the lawfulness we discovered contradicts elements of the current paradigm, here the parallel postulate. This lawfulness is thus not a higher principle in the current paradigm. We have now reached as far as we can go in the current paradigm.
- 3) In order to reach the third stage, we need to hold on to the idea of polarity – and shape a new paradigm around this lawfulness, making it into a higher principle in the new paradigm. An aspect of the old paradigm is rejected and the old paradigm is transformed into the new paradigm. In this example Euclidian geometry is transformed into projective geometry by rejecting the parallel postulate, which then leads to the plane at infinity being included as a geometrical element.

The example of the transition from Euclidean to projective geometry considered here illustrates what happens when we progress from the second to the third stage of Goethe's method: we change the paradigm. This example also shows that this transition is a *transformation* of the old paradigm, not its rejection. Amrine rightly notes that scientific revolution requires a new way of seeing and thereby the metamorphosis of the scientist. But if there is to be growth in science, this revolution also needs to be a metamorphosis of the paradigm.